8-3 Factoring $x^2 + bx + c$

Warm Up

1. Which pair of factors of 8 has a sum of 9? 1 and 8

2. Which pair of factors of 30 has a sum of −17? −2 and −15

Multiply.

3. (x +2)(x +3) $x^2 + 5x + 6$

4. $(r + 5)(r - 9)$ $r^2 - 4r - 45$

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Objective

Factor quadratic trinomials of the form $x^2 + bx + c$.

8-3 Factoring $x^2 + bx + c$

In Chapter 7, you learned how to multiply two binomials using the Distributive Property or the FOIL method. In this lesson, you will learn how to factor a trinomial into two binominals.

8-3 Factoring $x^2 + bx + c$

Notice that when you multiply $(x + 2)(x + 5)$, the constant term in the trinomial is the product of the constants in the binomials.

$$(x + 2)(x + 5) = x^2 + 7x + 10$$

You can use this fact to factor a trinomial into its binomial factors. Look for two numbers that are factors of the constant term in the trinomial. Write two binomials with those numbers, and then multiply to see if you are correct.

Example 1A: Factoring Trinomials by Guess and Check

Factor $x^2 + 15x + 36$ by guess and check.

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(x + \square)(x + \square)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

The constant term in the trinomial is 36.

$(x + 1)(x + 36) = x^2 + 37x + 36$ ✘

$(x + 2)(x + 18) = x^2 + 20x + 36$ ✘

$(x + 3)(x + 12) = x^2 + 15x + 36$ ✓

Try factors of 36 for the constant terms in the binomials.

The factors of $x^2 + 15x + 36$ are $(x + 3)(x + 12)$.

$$x^2 + 15x + 36 = (x + 3)(x + 12)$$

8-3 Factoring $x^2 + bx + c$

Remember!

When you multiply two binomials, multiply:

First terms

Outer terms

Inner terms

Last terms

Check It Out! Example 1a

Factor each trinomial by guess and check.

$x^2 + 10x + 24$

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(x + \square)(x + \square)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

The constant term in the trinomial is 24.

$(x + 1)(x + 24) = x^2 + 25x + 24$ ✗

$(x + 2)(x + 12) = x^2 + 14x + 24$ ✗

$(x + 3)(x + 8) = x^2 + 11x + 24$ ✗

$(x + 4)(x + 6) = x^2 + 10x + 24$ ✓

Try factors of 24 for the constant terms in the binomials.

The factors of $x^2 + 10x + 24$ are $(x + 4)(x + 6)$.

$$x^2 + 10x + 24 = (x + 4)(x + 6)$$

Check It Out! Example 1b

Factor each trinomial by guess and check.

$x^2 + 7x + 12$

$(\square + \square)(\square + \square)$ *Write two sets of Parentheses.*

$(x + \square)(x + \square)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

The constant term in the trinomial is 12.

$(x + 1)(x + 12) = x^2 + 13x + 12$ ✘ *Try factors of 12 for the constant terms in the binomials.*

$(x + 2)(x + 6) = x^2 + 8x + 12$ ✘

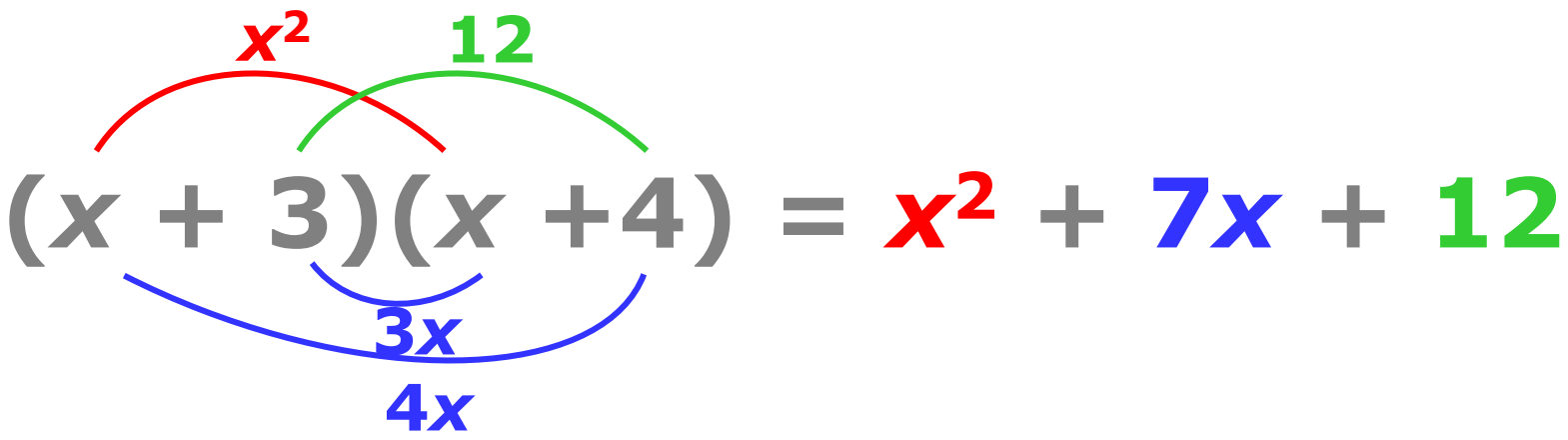
$(x + 3)(x + 4) = x^2 + 7x + 12$ ✓

The factors of $x^2 + 10x + 24$ are $(x + 4)(x + 6)$.

$$x^2 + 10x + 24 = (x + 4)(x + 6)$$

8-3 Factoring $x^2 + bx + c$

The guess and check method is usually not the most efficient method of factoring a trinomial. Look at the product of $(x + 3)$ and $(x + 4)$.



The coefficient of the middle term is the sum of 3 and 4. The third term is the product of 3 and 4.

8-3 Factoring $x^2 + bx + c$

Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.

Factors of 18	Sum	
1 and 18	19	✗
2 and 9	11	✗
3 and 6	9	✓

$x^2 + 9x + 18 = (x + 3)(x + 6)$

When c is positive, its factors have the same sign. The sign of b tells you whether the factors are positive or negative. When b is positive, the factors are positive and when b is negative, the factors are negative.

Example 2A: Factoring $x^2 + bx + c$ When c is Positive

Factor each trinomial. Check your answer.

$x^2 + 6x + 5$

$(x + \square)(x + \square)$

b = 6 and c = 5; look for factors of 5 whose sum is 6.

Factors of 5	Sum
1 and 5	6 ✓

The factors needed are 1 and 5.

$(x + 1)(x + 5)$

Check $(x + 1)(x + 5) = x^2 + x + 5x + 5$

$= x^2 + 6x + 5$ ✓

Use the FOIL method.

The product is the original polynomial.

Example 2B: Factoring $x^2 + bx + c$ When c is Positive

Factor each trinomial. Check your answer.

$x^2 + 6x + 9$

$(x + \square)(x + \square)$ *b = 6 and c = 9; look for factors of 9 whose sum is 6.*

Factors of 9	Sum
1 and 9	10 ✗
3 and 3	6 ✓

The factors needed are 3 and 3.

$(x + 3)(x + 3)$

Check $(x + 3)(x + 3) = x^2 + 3x + 3x + 9$ *Use the FOIL method.*

$= x^2 + 6x + 9$ ✓ *The product is the original polynomial.*

Example 2C: Factoring $x^2 + bx + c$ When c is Positive

Factor each trinomial. Check your answer.

$x^2 - 8x + 15$

$(x + \square)(x + \square)$

b = –8 and c = 15; look for factors of 15 whose sum is –8.

Factors of –15	Sum
–1 and –15	–16 ✗
–3 and –5	–8 ✓

The factors needed are –3 and –5 .

$(x - 3)(x - 5)$

Check $(x - 3)(x - 5) = x^2 - 3x - 5x + 15$ *Use the FOIL method.*

$= x^2 - 8x + 15$ ✓ *The product is the original polynomial.*

Check It Out! Example 2a

Factor each trinomial. Check your answer.

$x^2 + 8x + 12$

$(x + \square)(x + \square)$

b = 8 and c = 12; look for factors of 12 whose sum is 8.

Factors of 12	Sum
1 and 12	13 ✗
2 and 6	8 ✓

The factors needed are 2 and 6 .

$(x + 2)(x + 6)$

Check $(x + 2)(x + 6) = x^2 + 2x + 6x + 12$

$= x^2 + 8x + 12$ ✓

Use the FOIL method.

The product is the original polynomial.

Check It Out! Example 2b

Factor each trinomial. Check your answer.

$x^2 - 5x + 6$

$(x + \square)(x + \square)$

b = –5 and c = 6; look for factors of 6 whose sum is –5.

Factors of 6	Sum
–1 and –6	–7 ✗
–2 and –3	–5 ✓

The factors needed are –2 and –3.

$(x - 2)(x - 3)$

Check $(x - 2)(x - 3) = x^2 - 2x - 3x + 6$

$= x^2 - 5x + 6$ ✓

Use the FOIL method.

The product is the original polynomial.

Check It Out! Example 2c

Factor each trinomial. Check your answer.

$x^2 + 13x + 42$

$(x + \square)(x + \square)$

b = 13 and c = 42; look for factors of 42 whose sum is 13.

Factors of 42	Sum	
1 and 42	43	✗
2 and 21	23	✗
6 and 7	13	✓

The factors needed are 6 and 7.

$(x + 6)(x + 7)$

Check $(x + 6)(x + 7) = x^2 + 7x + 6x + 42$

Use the FOIL method.

$= x^2 + 13x + 42$ ✓

The product is the original polynomial.

Check It Out! Example 2d

Factor each trinomial. Check your answer.

$x^2 - 13x + 40$

$(x + \square)(x + \square)$

b = –13 and c = 40; look for factors of 40 whose sum is –13.

Factors of 40	Sum
–2 and –20	–22 ✗
–4 and –10	–14 ✗
–5 and –8	–13 ✓

The factors needed are –5 and –8.

$(x - 5)(x - 8)$

Check $(x - 5)(x - 8) = x^2 - 5x - 8x + 40$

$= x^2 - 13x + 40$ ✓

Use the FOIL method.

The product is the original polynomial.

When *c* is negative, its factors have opposite signs. The sign of *b* tells you which factor is positive and which is negative. The factor with the greater absolute value has the same sign as *b.*

Example 3A: Factoring $x^2 + bx + c$ When c is Negative

Factor each trinomial.

$x^2 + x - 20$

$(x + \square)(x + \square)$

b = 1 and c = –20; look for factors of –20 whose sum is 1. The factor with the greater absolute value is positive.

Factors of –20	Sum
–1 and 20	19 ✗
–2 and 10	8 ✗
–4 and 5	1 ✓

The factors needed are +5 and –4.

$(x - 4)(x + 5)$

Example 3B: Factoring $x^2 + bx + c$ When c is Negative

Factor each trinomial.

$x^2 - 3x - 18$

$(x + \square)(x + \square)$

b = –3 and c = –18; look for factors of –18 whose sum is –3. The factor with the greater absolute value is negative.

Factors of –18	Sum
1 and –18	–17 ✗
2 and – 9	– 7 ✗
3 and – 6	– 3 ✓

The factors needed are 3 and –6.

$(x - 6)(x + 3)$

Helpful Hint

If you have trouble remembering the rules for which factor is positive and which is negative, you can try all the factor pairs and check their sums.

Check It Out! Example 3a

Factor each trinomial. Check your answer.

$x^2 + 2x - 15$

$(x + \square)(x + \square)$

Factors of –15	Sum
–1 and 15	14 ✗
–3 and 5	2 ✓

$(x - 3)(x + 5)$

b = 2 and c = –15; look for factors of –15 whose sum is 2. The factor with the greater absolute value is positive.

The factors needed are –3 and 5.

Check It Out! Example 3b

Factor each trinomial. Check your answer.

$x^2 - 6x + 8$

$(x + \square)(x + \square)$

Factors of 8	Sum
–1 and –6	–7 ✗
–2 and –4	–6 ✓

b = –6 and c = 8; look for factors of 8 whose sum is –6.

The factors needed are –4 and –2.

$(x - 2)(x - 4)$

Check It Out! Example 3c

Factor each trinomial. Check your answer.

$x^2 - 8x - 20$

$(x + \square)(x + \square)$

Factors of –20	Sum
1 and –20	–19 ✗
2 and –10	–8 ✓

$(x - 10)(x + 2)$

b = –8 and c = –20; look for factors of –20 whose sum is –8. The factor with the greater absolute value is negative.

The factors needed are –10 and 2.

8-3 Factoring $x^2 + bx + c$

A polynomial and the factored form of the polynomial are equivalent expressions. When you evaluate these two expressions for the same value of the variable, the results are the same.

Example 4A: Evaluating Polynomials

Factor $y^2 + 10y + 21$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

$y^2 + 10y + 21$

$(y + \square)(y + \square)$

b = 10 and c = 21; look for factors of 21 whose sum is 10.

Factors of 21	Sum
1 and 21	7 ✗
3 and 7	10 ✓

The factors needed are 3 and 7.

$(y + 3)(y + 7)$

Example 4A Continued

Evaluate the original polynomial and the factored form for $n = 0, 1, 2, 3,$ and 4.

y	$(y + 7)(y + 3)$
0	$(0 + 7)(0 + 3) = 21$
1	$(1 + 7)(1 + 3) = 32$
2	$(2 + 7)(2 + 3) = 45$
3	$(3 + 7)(3 + 3) = 60$
4	$(4 + 7)(4 + 3) = 77$

y	$y^2 + 10y + 21$
0	$0^2 + 10(0) + 21 = 21$
1	$1^2 + 10(1) + 21 = 32$
2	$2^2 + 10(2) + 21 = 45$
3	$3^2 + 10(3) + 21 = 60$
4	$4^2 + 10(4) + 21 = 77$

The original polynomial and the factored form have the same value for the given values of n.

Check It Out! Example 4

Factor $n^2 - 7n + 10$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

$n^2 - 7n + 10$

$(n + \blacksquare)(n + \blacksquare)$

b = –7 and c = 10; look for factors of 10 whose sum is –7.

Factors of 10	Sum
–1 and –10	–11 ✗
–2 and –5	–7 ✓

The factors needed are –2 and –5.

$(n - 5)(n - 2)$

Check It Out! Example 4 Continued

Evaluate the original polynomial and the factored form for $n = 0, 1, 2, 3,$ and 4.

n	$(n - 5)(n - 2)$
0	$(0 - 5)(0 - 2) = 10$
1	$(1 - 5)(1 - 2) = 4$
2	$(2 - 5)(2 - 2) = 0$
3	$(3 - 5)(3 - 2) = -2$
4	$(4 - 5)(4 - 2) = -2$

y	$n^2 - 7n + 10$
0	$0^2 - 7(0) + 10 = 10$
1	$1^2 - 7(1) + 10 = 4$
2	$2^2 - 7(2) + 10 = 0$
3	$3^2 - 7(3) + 10 = -2$
4	$4^2 - 7(4) + 10 = -2$

The original polynomial and the factored form have the same value for the given values of n.

Lesson Quiz: Part I

Factor each trinomial.

1. $x^2 - 11x + 30$ $(x - 5)(x - 6)$

2. $x^2 + 10x + 9$ $(x + 1)(x + 9)$

3. $x^2 - 6x - 27$ $(x - 9)(x + 3)$

4. $x^2 + 14x - 32$ $(x + 16)(x - 2)$

Lesson Quiz: Part II

Factor $n^2 + n - 6$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$,and 4.

$(n + 3)(n - 2)$

n	$n^2 + n - 6$
0	$0^2 + 0 - 6 = -6$
1	$1^2 + 1 - 6 = -4$
2	$2^2 + 2 - 6 = 0$
3	$3^2 + 3 - 6 = 6$
4	$4^2 + 4 - 6 = 14$

n	$(n + 3)(n - 2)$
0	$(0 + 3)(0 - 2) = -6$
1	$(1 + 3)(1 - 2) = -4$
2	$(2 + 3)(2 - 2) = 0$
3	$(3 + 3)(3 - 2) = 6$
4	$(4 + 3)(4 - 2) = 14$